

Fluid Flow In 2-D Petroleum Reservoir Using Darcy's Equation

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1. Abstract

The purpose of this paper is to test the Darcy's Equation and investigate, by simulations, how it's suitable to use in one-phase oil reservoir. To be used later in history matching procedure.

The model used is a flow in a saturated reservoir and this model similar to steady state two-dimensional (2-D) saturated porous media.

Reservoir engineering is based on the understanding of fluid flow in porous media. Our aim to produce reliable code in matlab is relevant for the reservoir process (as simulator). To be used instead of traditional simulator, Black Oil Applied Simulator Tools (BOAST) which is written in fortran, and widely used in almost works concern reservoir simulation.

Key words: Partial Differential Equation, Finite Difference, Reservoir, Simulation, Matlab.

2. Introduction

Reservoir engineering is based on the understanding of fluid flow in porous media. We must have some data about permeability, porosity, saturation, and relative permeability for oil, for a range of process conditions. We used fluid flow model in 2-D: flow in a porous media. The model used is a flow in a saturated reservoir and this model similar to steady state 2-D saturated porous media. The reservoir flow uses a potential formulation, and we apply this model to reservoir management (Landa, et al, 2000).

The reservoir modeling is a complex, multidisciplinary task. Once satisfactorily accomplished, the resulting model is used by operators and other interested parties for predicting performance under the range of operating and maintenance scenarios, for planning development strategies and for assisting production operations. (Parish, et al., 1993)

Numerical simulation is widely used for predicting reservoir behavior and forecasting its performance. However, the mathematical model used in the simulation requires the knowledge of subsurface properties. Since petroleum reservoirs are relatively inaccessible for sampling, the measurable quantities at the well provide the essential information for reservoir description (Ewing, et. al., 1995).

The obtained data at the sampling locations can be divided into two categories: static and dynamic. Static data such as permeability and porosity do not evolve considerably during the reservoir lifetime. This can be considered the intrinsic identity of the reservoir. On the other hand, dynamic data obtained at wells, such as hydrocarbon production and pressure history, are changing and observable reservoir response resulting from human and/or perturbation, ultimately. These two categories of data lead to two different approaches for assessing reservoir properties.

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The first approach, commonly known as geometrical reservoir characterization, focuses on static data. This approach uses the spatial correlation of static data to predict the unknown parameters at unsampled locations. In the second approach, subsurface properties are estimated through an inverse modeling procedure, which matches the dynamic data, by comparing simulated production or pressure, to field data. Note: inverse modeling takes into account the fluid flow when assessing reservoir properties. Until now both approaches were used only for predicting geological models, while little attention was given to other reservoir engineering parameters. Moreover, geostatistical methods becomes a major research area after the mid 1980's. Consequently, research in inverse modeling and automatic history matching lost its late 1970's and early 1980's vigor with few exceptions. Although a number of algorithms have been proposed before the mid 1980's, automatic history matching has not found widespread use yet. A major reason was the use of deterministic optimization methods, which can handle a limited number of parameters.

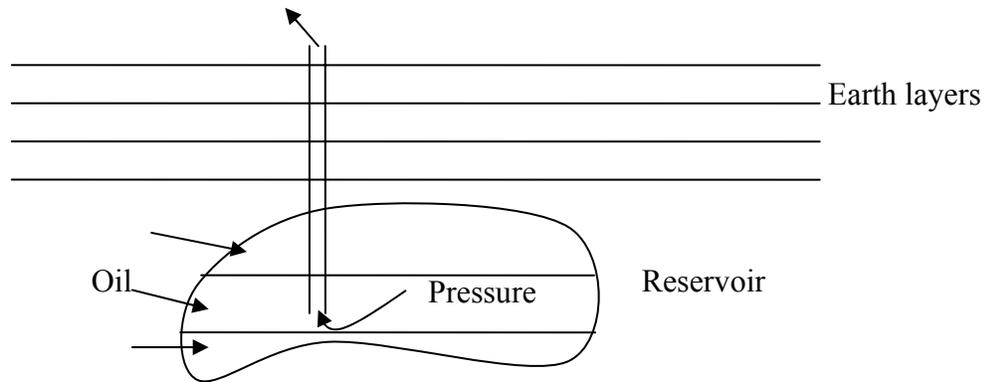


Figure 1. Reservoir description

2. Previous Works

History matching is formulated depending on the process of determining unknown parameter values for a mathematical reservoir model, such as permeability and porosity, which give the closest fit of measured and calculated pressures. In principle, one would like an automatic routine for history matching, applicable to simulators of varying complexity, one that does not achieve a set of parameter estimates.

In the seventh decade of this century, various automatic and semiautomatic history matching techniques have been introduced. Jacquard and Jain (1965) presented a technique based on a version of the method of steepest descent. They did not consider their method to be fully operational, however, due to the lack of experience with convergence. Jahns (1965) presented a method based on the Gauss-Newton equation with a stepwise solution for speeding the convergence; but his procedure still required a large number of reservoir simulation to lead to a solution. Coats et al. (1970) presented a workable automatic history matching procedure based on least-squares and linear programming. Slater and Durrer (1971) presented a method based on a gradient method and linear programming. In their study they mentioned the difficulty of choosing a step size for their gradient method, especially for problems involving low values of porosity and permeability. They also pointed out the need for a fairly small range in their reservoir description parameters for highly non-linear problems.

Thomas et al. (1972) presented a non-linear optimization technique that automatically varies reservoir performance. Their method based on the classical Gauss-Newton least-squares procedure. The method is a non-linear algorithm that will match both linear and non-linear systems in reasonable number of simulations. Wasserman et al. (1975) applied the material presented by two groups of scientists Chen et al. (1974) and Chavent et al. (1975) to practical reservoir problems. The pressure history matching algorithm used was initially based on a discretized single-phase reservoir model. Multiphase effects are approximately treated in the single-phase model by multiplying the transmissibility and storage terms by saturation-simulator run. Thus, all the history matching is performed by a “pseudo” single-phase model. The multiplicative factors for transmissibility and storage are updated when necessary. The matching technique can change any model permeability thickness value.

Dogru et al. (1977) presented methods of non-linear regression theory, which was applied to the reservoir history matching problem to determine the effect of erroneous parameter estimates obtained from well testing on the future prediction of reservoir pressures. Several studies on history matching have indicated that the well-test approach for determining the reservoir parameters often suffers from incorrect and nonunique parameter estimates. The factors that affect the parameter estimation can be classified as model errors, observability, measurement errors or noise, history time, test procedure, and optimization procedure.

Watson, et al. (1980) cleared that the aspect of the reservoir history matching problem that distinguishes it from other parameter estimation problems in science and engineering is the large dimensionality of both system state and the unknown parameters. As a result of this large dimensionality, computational efficiency becomes a prime consideration in the implementation of an automatic history matching method. In all parameter estimation methods, a tradeoff exists between iteration and the speed of convergence of the method. An important saving in computing time was realized in single-phase automatic history matching through the introduction of optimal control theory as a method for calculating the gradient of the objective function with respect to the unknown parameters. This technique currently is limited to first-order gradient methods. First-order gradient methods generally converge more slowly than those of higher order do.

Lee et al. (1986) presented an algorithm for an automatic history matching which developed from spline approximations of permeability and porosity distributions and from theory of regularization to estimate permeability or porosity in a 1-Phase, 2-D areal reservoir from well pressure data. The algorithm uses conjugate gradient method as its core minimization method. A number of numerical experiments are carried out to evaluate the performance of the algorithm. Comparisons with conventional (non-regularized) automatic history matching algorithms indicate the superiority of the new algorithm with respect to the parameter estimates obtained.

3. What is Reservoir Fluids?

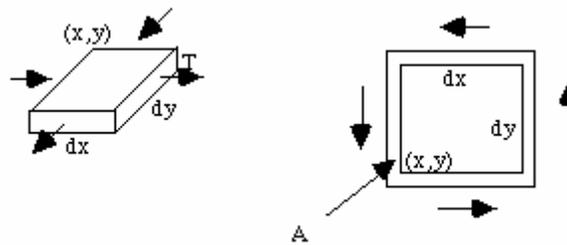
The velocity of the fluid in reservoir has the form $(u(x,y), (x,y), 0)$. In other words, this means it is a 2-D steady state fluid flow. It is useful to be able to give a mathematical description of reservoir fluid.

The compressibility of the fluid can be quantified by the divergence of the velocity. In 2-D the divergence of (u,v) is $u_x + v_y$. This measures how much mass enters a small volume in a given unit of time. To understand this, consider the small thin rectangular mass with density ρ and discretize $u_x + v_y$

Flow in and out of volume

$$(dx dy T) = \rho T dy (u(x+dx, y) - u(x,y)) dt + \rho T dx (v(x, y+dy) - v(x,y)) dt$$

Divide by $(dx dy T)dt$ and let dx and dy go to zero to get rate of change of mass per unit volume is $\rho(u_x + v_y)$.



$$v_x + u_y = 0$$

Incompressible

$$v_x - u_y = 0$$

Irrotational

Figure2. Incompressible and irrotational 2-D fluid.

A fluid with no circulation or rotation can be described by the curl of the velocity vector. In 2-D the curl of (u, v) is $v_x - u_y$. Also the discrete form of this gives some insight to the meaning of this. The circulation or momentum of the loop about the volume $(dx dy T)$ with cross sectional area A and density ρ is

$$\text{momentum} = \rho A dy (v(x+dx, y) - v(x,y)) - \rho A dx (u(x, y+dy) - u(x,y)).$$

Divide by $\rho (A dy dx)$ and let dx and dy go to zero to get $v_x - u_y = 0$ for no rotation.

4. Applied to Saturated 2-D 1-Phase Reservoir

Consider a saturated reservoir which is to have at least one well. Assumed the region is in the xy -plane and that the oil moves towards the well in such a way that the velocity vector is in the xy -plane. At the top and bottom of the xy region we will assume there is no flow through these boundaries. However, assume there is a wide supply from the left and right boundaries so that the pressure is fixed. The problem is to determine the oil flow rates of well, location of well and number of wells so that there is still oil to be pumped out.

If a cell does not contain a well and is in the interior, then $u_x + v_y = 0$. If there is a well in a cell, then $u_x + v_y < 0$. The motion of the fluid is governed by an empirical law which is analogous to the Fourier heat law.

$$\text{Darcy's Law. } (u,v) = -K(h_x, h_y)$$

where

h is the hydraulic head pressure and

K is the hydraulic conductivity which is constant for saturated regions.

So, we have $u_x + v_y = -(Kh_x)_x - (Kh_y)_y$ is zero or negative.

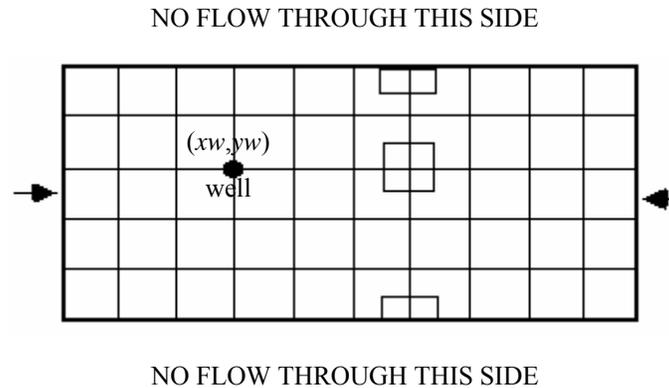


Figure 3. 2-D 1-Phase reservoir flow in porous media.

5. Model

The model have a partial differential equation similar to that of the 2-D heat diffusion model, with have different boundary conditions. For fluid flow reservoir problems, they are either a given function along part of the boundary, or they are a zero derivative for the remainder of the boundary.

6. Fluid Flow Reservoir Model

$$-(Kh_x)_x - (Kh_y)_y = \begin{cases} 0 & , (x,y) \notin \text{well} \\ -R & , (x,y) \in \text{well} \end{cases} \quad (x,y) \in (0,y) (0,L) \times (0,H)$$

$$Kh_y = 0 \text{ for } y=0 \text{ and } y=H, \text{ and}$$

$$h_x = h_0 \text{ for } x = 0 \text{ and } x = L.$$

7. Problem Treatment

In this problem the finite difference method used coupled with the SOR iterative method. For the $(dx \ dy)$ cells in the interior. For the portions of the boundary where the derivative is set equal zero on a half cell $(dx/2 \ dy)$ or $(dx \ dy/2)$, some additional code inserted inside the SOR loop. For example, notice the model where $h_y = 0$ at $y = H$ on the half cell $(dx \ dy/2)$. The finite difference equation and corresponding line of SOR code are, respectively, $u = h$:

$$-[(u(i+1,j) - u(i,j))/dx - (u(i,j) - u(i-1,j))/dx]/dx - [(0) - (u(i,j) - u(i,j-1))/dy]/(dy/2) = 0$$

$$utemp = ((u(i+1,j) + u(i-1,j))/(dx*dx) + 2*u(i,j-1)/(dy*dy))/(2/(dx*dx) + 2/(dy*dy)).$$

$$u(i,j) = (1 - w)* u(i,j) + w*utemp.$$

In the following implementations observe where the extra lines of code are that reflect these derivative boundary conditions.

8. Implementation.

The fluid flow reservoir model uses the following parameters:

$$\begin{array}{llll} L = 5,000 & dx = h = 100 & xw = (iw-1)h & h\infty = 100 \\ H = 1,000 & dy = h = 100 & yw = (jw-1)h & K = 10. \end{array}$$

A single well with a flow rate of -1000 was used in the first numerical experiment. The first output graphs are plots of the hydraulic head pressure as a function of x and y . Note that the pressure near the well has dropped from 100 to about 30. The second experiment has two wells with the same flow rate. In this case the pressures are negative near both wells. This indicates that before any steady state solution was achieved, the wells went dry!

9. Matlab Code for Fluid Flow in 2-D 1-Phase Saturated Reservoir in Porous Media.

```
clear all
K=10;
well=-1000;
iw=16;
jw=6;
eps=0.0001;
nx= 50;
ny=10;
H=1000;
w=1.7;

u=ones(nx+1,ny+1)*100;

h=H/ny;

maxit=400;
tol=eps*h*h;
for m =1 : maxit
    numi=0;
    j= 1;
    for i = 2 :nx
        utemp= (( 2*u(i,j+1) + u(i+1,j) + u(i-1,j))*0.25);
        utemp= (1-w)*u(i,j) + w*utemp;
        error= abs(utemp - u(i,j));
        u(i,j)=utemp;
        if (error < tol)
            numi=numi +1;
        end
    end
end
for j = 2 : ny
    for i = 2 : nx
        utemp= (( u(i,j-1) + u(i-1,j) + u(i+1,j) + u(i,j+1))*0.25);
```

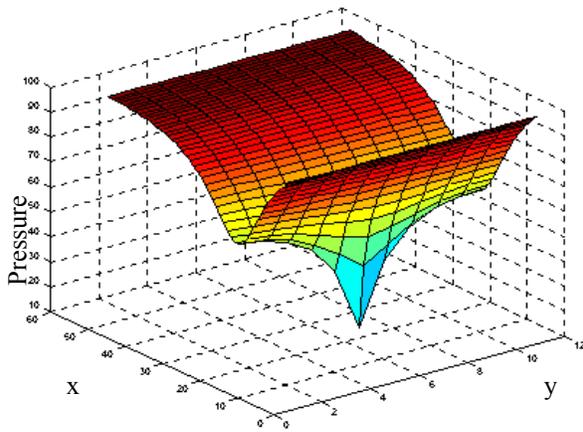
```

        if ((i==iw) & (j==jw))
            utemp= (( u(i,j-1) + u(i-1,j) + u(i+1,j) + u(i,j+1) +
well/K)*0.25);
        end
        utemp= (1-w)*u(i,j) + w*utemp;
        error= abs(utemp - u(i,j));
        u(i,j)=utemp;
        if (error < tol)
            numi=numi +1;
        end
    end
end
j= ny + 1;
for i = 2 : nx
    utemp= (( 2*u(i,j-1) + u(i+1,j) + u(i-1,j))/4);
    utemp= (1-w)*u(i,j) + w*utemp;
    error= abs(utemp - u(i,j));
    u(i,j)=utemp;
    if (error < tol)
        numi=numi +1;
    end
end
end

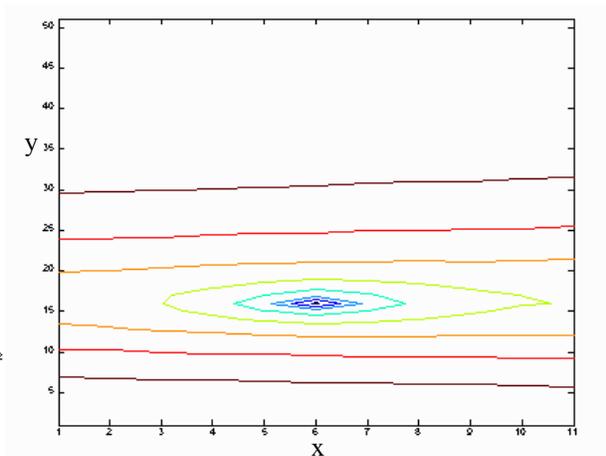
if (numi == (nx-1)*(ny+1)) break; end
end

> darcy2d
> m
> m      27
> numi
> numi   539
surf(u)
contour(u)

```



a. Surf 3-D

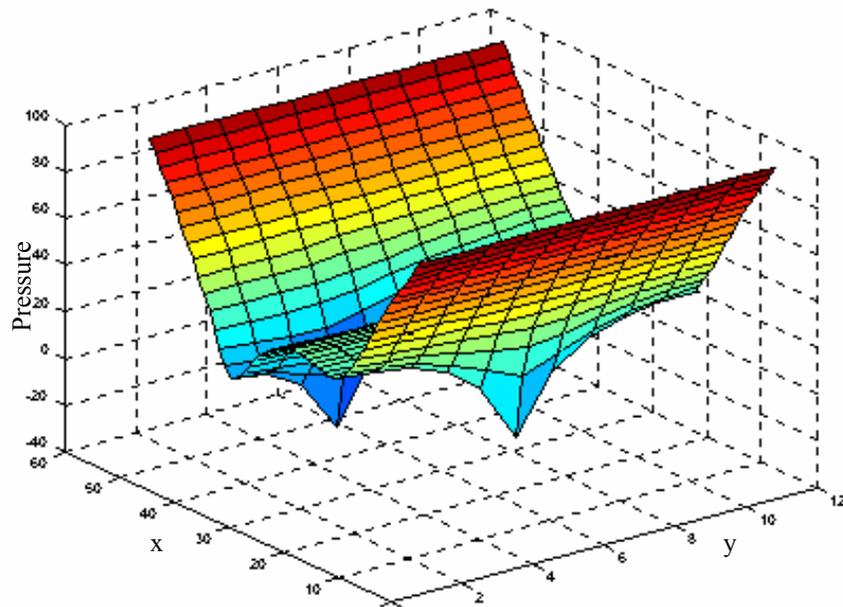


b. Contour 2-D

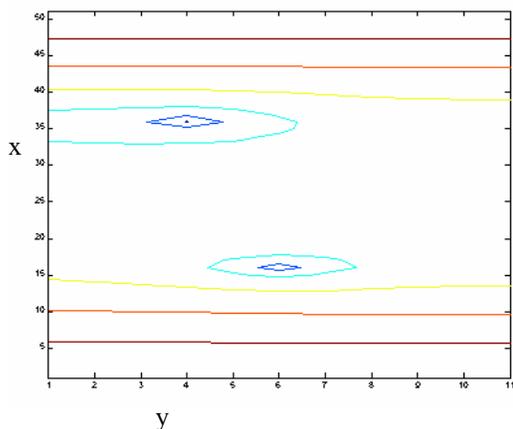
Figure 4. Well at (16,6) with flow Rate 1000.

9. Result Analysis

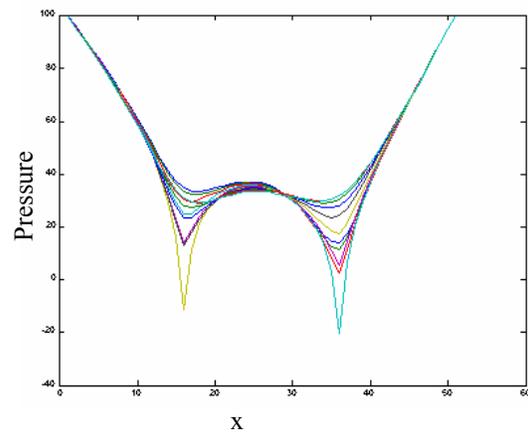
A two-dimensional, black-oil reservoir was modeled as shown in Figure 4. In this instance, to avoid the effects of the heterogeneity in understanding the problem from the conceptual point of view, the permeability and porosity were set constant throughout the reservoir. In this model, well (16,6) is producing at constant flow rates of 1000. Figure 5 also shows the pressure in the reservoir as a function of time if we have two wells production (16,6) and (36,4) with flow rates of 1000. The pressure maps are shown here to illustrate what is going on in the reservoir and to help understand the results of the analysis.



a. Surf 3-D



b. Contour 2-D



c. Plot 2-D

Figure 5. Wells at (16,6) and (36,4) with flow rates of 1000.

10. Assessment

This model has enough assumptions to rule out many real applications. For oil reservoir problems the soils are usually not fully saturated, and the hydraulic conductivity can be highly nonlinear and can vary with space according to the soil types. Often the soils are very heterogeneous, and the soil properties are unknown. The model may require 3-D calculations and irregular shaped domains. Fluid flow may often be compressible and irrotational. The good news is that the more complicated models have many subproblems which are similar to model from heat diffusion, fluid flow in saturated 2-D 1-Phase oil reservoir in porous media.

11. Conclusions

The main purpose of this work was to develop procedure for Darcy's equation for fluid flow. A second objective was to develop procedure to use as a simulator of reservoir to assess the reservoir behavior.

The research resulted in the development of a procedure that allows us to integrate information from several sources to determine the pressure of reservoir.

The components of the procedure developed during this work included:

- A numerical reservoir simulator with the capability of computing the pressure as part of the solution process. The simulator was limited to two-dimensional reservoirs and oil system. We can expand the approach to three dimensions and to gas-oil-water systems.
- Mathematical procedures to construct dynamic reservoir objects.
- Using the finite difference algorithm to discretize the model which in form of a partial differential equation.

The example of reservoir objects shown here were relatively simple, but the method is applicable to more complex cases. The method to compute pressure, as developed in this work, is relatively simple to implement when it is possible to have access to the numerical reservoir simulator computer code. Thus the complexity of modeling the reservoir became easy because it done in the simulation part.

From the results of the numerical experiments performed during this research it is possible to conclude that the method can be utilized as a tool for reservoir characterization. The method seems to be more useful when the dynamic data set consists of a large number of measurements that are difficult to honor with the existing geostatistical approaches; this may be the case in mature fields that have already gone through a secondary recovery process and in which data have been gathered over many years.

For further research, one of the main assumptions in this work is that the mathematical model (numerical reservoir simulator) can perform accurate predictions. Thus more research has to be conducted in the areas of modeling, especially in the area of well modeling in multiphase regimes, and in the area of controlling numerical dispersion in finite difference models.

References

- Chavent, G., Dupuy, M., and Lemonnier, P., 1975; "History-Matching by Use of Optimal Control Theory", SPE Journal, 259, 74-86.
- Chen W. H., Gavalas, G. R., Seinfeld, J. H., and Wasserman, M. L., 1974; "A New Algorithm for Automatic History Matching", SPE Journal, 257, 593-608.
- Coats, K. H., Dempsey, J. R., and Henderson, J. H.; 1970, "A new Technique for Determining Reservoir Description from Field Performance Data", SPE Journal, 249, 66-74.
- Dogru, A. H., Dixon, T. N., and Edgar, T. F., 1977; "Confidence Limits on the Parameters Slightly Compressible, Single-phase Reservoirs", SPE Journal, 42-56.
- Ewing R.E., Pilant M.S., Wade J.G., and Watson A.T, 1995, Identification and Control Problems in Petroleum and Groundwater Modeling, Department of Mathematics and Institute for Scientific Computation, Texas A&M University
- Ewing R.E., Pilant M.S., Wade J.G., and Watson A.T., 1994, Parameter Estimation in Petroleum and Groundwater Modeling, Department of Mathematics and Institute for Scientific Computation, Texas A&M University
- Jacquard, P., and Jain, C.; 1965, "Permeability Distributions from Field Pressure Data", SPE Journal, 234, 281-294.
- Jahns, H. O.; 1965, "A Rapid Method for Obtaining a Pressure Response Data", SPE Journal, 237, 315-327.
- Landa J. L, Horne R.N, M.M. Kamal, and C.D. Jenkins, 2000, Reservoir Characterization Constrained to Well-Test Data: A Field Example, Arco Exploration and Production Technology
- Landa J. L., 1997, Reservoir Parameter Estimation Constrained to Pressure Transients, Performance History and Distributed Saturation Data, Dissertation, Doctor of Philosophy, Stanford University
- Langaas K., Nilsson S., Stavland A., 2002, Pore-Scale Simulations of Dpr Gel Placement, Rf-Rogaland Research, Norway
- Lee, T., Kavaris, C., and Seinfeld, J. H., 1986; "History-Matching by Spline Approximation and Regularization in Single-Phase Areal Reservoirs", SPE Reservoir Engineering Journal, Sept. 521-543.
- Parish, R. G., Calderbank, V. J., Watkins, A. J., Muggeridge, A. H., Goode, A. T., and Robinson, P. R., 1993; "Effective History-Matching: The Application of Advanced Software Techniques to the History-Matching Process", SPE Technical Papers 25250.
- Seidman T. I., 1996, Control of The Heat Equation, Department of Mathematics and Statistics, University of Maryland Baltimore County, Baltimore, USA.
- Slater, G. E., and Durrer, E. J.; 1971, "Adjustment of Reservoir Simulation Models to Match Field Performance", SPE Journal, 251, 259-305.
- Thomas, L. K., Hellums, L. J., and Reheis, G. M.; 1972, "A Nonlinear Automatic History Matching Technique for Reservoir Simulation Model", SPE Journal, 253.
- Wasserman, M. L., Emanuel, A. S., and Seinfeld, J. H., 1975; "Practical Applications of Optimal Control Theory to History-Matching Multiphase Simulator Reservoir Models", SPE Journal, 259, 347-355.
- Watson, A. T., Seinfeld, J. H., Gavalas, G. R., and Woo, P. T., 1980; "History-Matching in Two-Phase Petroleum Reservoirs", SPE Journal, Dec., 521-532.